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by

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In this effort we further studied the application of the higher order crossings on stationary time series and homogeneous random fields. The Higher Order Crossings Theorem has been extended to homogeneous random fields [?] and the ψ^2 first introduced in the literature in [4] has been further studied. In particular, since the Higher Order Crossings Theorem can be extended to random fields this suggests that ψ^2 can be used in texture discrimination which is indeed the case.

We have studied the power of ψ^2 via simulation techniques of series and fields. As the alternative becomes more distinguishable so is the increase in the power as expected. We still know very little about the eigenvalues of ψ^2 although we have computed some for special cases.

Another aspect we have been studying is graphics in time series analysis [1]. There is no theory of graphics as such, but we were able to prove various graphical results by [illegible] a series by a random curve. In this connection we mention our work [2] in which the notion of a unit has been advanced in connection with the appearance of binary sequences.

[illegible] INGS RESULTS

1. Let $\{Z(t_1,t_2),\ t_1,t_2 = 0, \pm 1, \pm 2, \cdots\}$ be a zero mean stationary random field. Then if H is the clipping operator and ∇ is the difference operator we have

$$\{H\nabla^k_{t_1}\nabla^k_{t_2}Z(t_1,t_2)\} \Rightarrow \{\underset{\sim}{a},\underset{\sim}{a}'\}, \quad k \to \infty,$$

where

$$\underset{\sim}{a} = \begin{array}{cccccc} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots 1 & 0 & 1 & 0 & 1 & 0 \cdots \\ \cdots 0 & 1 & 0 & 1 & 0 & 1 \cdots \\ \cdots 1 & 0 & 1 & 0 & 1 & 0 \cdots \\ \cdots 0 & 1 & 0 & 1 & 0 & 1 \cdots \\ \cdots 1 & 0 & 1 & 0 & 1 & 0 \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

and $\underset{\sim}{a}'$ is the same array shifted to the right.

2. Let $D_{j,N}$ be the total number of symbol changes in the binary array $H\nabla^j_{t_1}\nabla^j_{t_2}Z(t_1,t_2)$. Then

a) $\lim\limits_{j\to\infty}\lim\limits_{N\to\infty} \dfrac{D_{j,N}}{N^2} = 1$

b) $\lim\limits_{N\to\infty} \dfrac{D_{j,N}}{N^2}$ is almost surely increasing in j.

3. Let $\hat{p}_j = D_j/(N-1)$, $p_j = E\hat{p}_j$, $\hat{p} = (\hat{p}_1,\cdots,\hat{p}_{k-1})'$

where $\Delta_j = D_j - D_{j-1}$ as in [5]. Define

$$\underset{\sim}{P} = D_{AG}(p_1, \cdots, p_{k-1}) - \underset{\sim}{p}\,\underset{\sim}{p}'.$$

Then

$$\psi_N^2 = (N-1)(\hat{\underset{\sim}{p}}-\underset{\sim}{p})'\underset{\sim}{P}^{-1}(\hat{\underset{\sim}{p}}-\underset{\sim}{p})$$

and under regularity conditions

$$\sqrt{N-1}\,(\hat{\underset{\sim}{p}}-\underset{\sim}{p}) \to N(\underset{\sim}{0},\underset{\sim}{V}), \quad N \to \infty.$$

Then

$$\psi_N^2 \to \sum_{j=1}^{K-1} \delta_j Z_j^2, \quad Z_j \sim N(0,1) \text{ INDEP.}$$

And

$$\bar{\delta} = \frac{1}{K-1} \sum_{j=1}^{K} \frac{V_{jj}}{p_j}$$

It follows that in order to study the distribution of ψ_N^2, a great deal can be learnt from the asymptotic variance of $\Delta j/\sqrt{N-1}$.

4. From many simulations, it can be shown that 28 is a critical value of ψ^2 which roughly corresponds to [illegible]. Our power calculations follow using this critical value.

TRUE H_1: ARMA(2,2)				FALSE H_0: AR(1)	POWER
φ_1	φ_2	θ_1	θ_2	φ	$p(U \geq 28)$
0.2	0.0	0.8	0.0	0.6	0.993
0.3	0.0	0.6	0.0	0.62	0.920
0.4	0.4	0.0	0.0	0.66	0.874
0.5	0.0	0.5	0.0	0.72	0.[illegible]06
0.0	0.0	0.6	0.0	0.45	0.583
0.6	0.3	0.0	0.0	0.85	0.540
0.6	0.0	0.3	0.0	0.73	0.378
0.6	0.3	0.5	0.2	0.95	0.464
0.0	0.0	0.5	0.5	0.51	0.13[illegible]
0.8	0.0	0.0	0.0	0.85	0.224
-0.6	0.0	-0.5	0.0	-0.65	0.194
0.0	0.0	0.3	0.0	0.29	0.082
0.0	0.0	0.0	0.0	0.013	0.046

Similar results hold for random fields with critical value 65.

II. RESULTS FOR GRAPHICS

Assume $|Z_t| < A$, and in $(-A,A)$ we define a uniform process $\{U_t\}$ independent of $\{Z\}$ and made of independent variates. We have two time series

$$Z_1\ Z_2\ \cdots\ Z_N \leftarrow \text{Time Series}$$

$$U_1\ U_2\ \cdots\ U_N \leftarrow \text{"Random Curve"}$$

1. The finite dimensional distributions of a Gaussian process are completely determined by graphical features.

2. The variance of a rounded stationary process can be obtained by counting the number of crossings by Z_t of the random curve, the number of sojourns of at least two time periods above and below the random curve, and the number of crossings of ∇Z_t of the corresponding random curve.

3. Let $\{U_t^{(k)}\}$ be the random curve which corresponds to $\{\nabla^k Z_t\}$. The crossings of the $U^{(k)}$-curve by $\nabla^k Z_t$ completely determine the covariance structure of $\{Z_t\}$.

4. Let $Z_t = \phi Z_{t-1} + a_t$ where $|\phi| < 1$ and $a_t \sim$ Cauchy with characteristic function $\exp[-(1-|\phi|)|\theta|]$. Then Z_t is Cauchy with characteristic function $\exp[-|\theta|]$. Note, the moments of Z_t do not exist. Yet ϕ is estimable from the axis crossings by Z_t, $t = 1,\cdots,N$!

III. m'th ORDER UNITS FOR BINARY SEQUENCES

An m'th order unit of a binary sequence, is a subsequence which starts with 1, ends with m separating 0's and in which each 0-run consists of at most m - [illegible]

In [?] we have various combinatorial results. They lead to: $I_K\{X_t\}$ is m'th order Markov and $S = \sum_1^N X_t$, X_t being binary, then

$$P(\xi=s) \sim \left(\begin{array}{l}\text{\# of permutations of the}\\ \text{m'th order units with the}\\ \text{free 0's}\end{array}\right) p^{s}_{\underbrace{10\cdots0}_{m}}\; p^{n-s-m}_{\underbrace{00\cdots0}_{m+1}}$$

$$\rightarrow e^{-\beta}\beta^{s}/s!$$

where $\beta = Np_{100\cdots0}$ and $p_{10\cdots0} = p(1|0\cdots0)$, $p_{00\cdots0} = p(0|0\cdots0)$.

Our combinatorial results help to define higher order degeneracies of particles in the physical sense.

References

[1] Kedem, B.(1980). Graphical Considerations in Time Series Analysis, University of Maryland Report MD-80-43-BK (Submitted).

[2] Kedem, B.(1981). On Nearest Neighbor Degeneracies of Indistinguishable Particles, <u>Journal of Math. Physics</u>, 22, 456-461.

[3] Kedem, B.(1981). A Convergence Phenomenon and Nonparametric Classification of Random Fields. Submitted for publication.

[4] Kedem, B. and Slud, E. (1981). On Goodness of Fit of Time Series Models, <u>Biometrica</u>, 68, 551-556.

[5] Kedem, B. and Slud, E. (1979). Higher Order Crossings in the Discrimination of Time Series, I, II, Univ. of Maryland Tech. Reports MD79-73-BK/ES, MD-79-89-BK/ES.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this effort further study was made of the implication of the higher order crossings on stationary time series and homogeneous random fields. The Higher Order Crossings Theorem has been extended to homogeneous random fields and the psi-square first introduced in the literature by the principal investigator has been further studied. In particular, since the Higher Order Crossings Theorem can be extended to random fields this suggests that psi-square can be used in texture discrimination which is indeed the case. The (CONT.)

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investigators have studied the power of psi-square via simulation techniques of series and fields. As the alternative becomes more distinguishable so is the increase in the power as expected. Another aspect studied is graphics in the series analysis. There is no theory of graphics as such, but the investigators were able to prove various graphical results by clipping a series by a random curve. In this connection the investigators did some work in which the notion of a unit has been advanced in connection with the appearance of binary sequences.